

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions		M.M., Min.
Single choice Objective (no negative marking) Q.1,2,3,4	(3 marks, 3 min.)	[12, 12]
Subjective Questions (no negative marking) Q.5,6,7	(4 marks, 5 min.)	[12, 15]
Match the Following (no negative marking) Q.8	(8 marks, 8 min.)	[8, 8]

- Range of the function  $f(x) = \frac{\ell nx}{\sqrt{x}}$  is  
 (A)  $(-\infty, e)$                       (B)  $(-\infty, e^2)$                       (C)  $\left(-\infty, \frac{2}{e}\right]$                       (D)  $\left(-\infty, \frac{1}{e}\right]$
- Let  $\tan(2\pi|\sin\theta|) = \cot(2\pi|\cos\theta|)$ , where  $\theta \in \mathbb{R}$  and  $f(x) = (|\sin\theta| + |\cos\theta|)^x$ ,  $x \geq 1$ . Then range of  $f(x)$  does not include  
 (A) 1                      (B) 2                      (C) 3                      (D) 4
- Range of the function  $f(x) = \sqrt{\sin^{-1}|\sin x| - \cos^{-1}|\cos x|}$  is  
 (A)  $\{0\}$                       (B)  $[0, \sqrt{\pi/2}]$                       (C)  $[0, \sqrt{\pi}]$                       (D) none of these
- If  $f(4) = g(4) = 2$ ,  $f'(4) = 9$ ,  $g'(4) = 6$ , then  $\lim_{x \rightarrow 4} \frac{\sqrt{f(x)} - \sqrt{g(x)}}{\sqrt{x} - 2}$  is equal to  
 (A)  $3\sqrt{2}$                       (B)  $\frac{3}{\sqrt{2}}$                       (C) 0                      (D) does not exist
- Evaluate :  
 (i)  $\lim_{x \rightarrow \infty} \frac{\cot^{-1}(\sqrt{x+1} - \sqrt{x})}{\sec^{-1}\left(\frac{2x+1}{x-1}\right)^x}$                       (ii)  $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r}{1+r^2+r^4}$
- Let  $f(x) = \begin{cases} \frac{\sin ax^2}{x^2}; & x \neq 0 \\ \frac{3}{4} + \frac{1}{4a}; & x = 0 \end{cases}$ . For what values of  $a$ ,  $f(x)$  is continuous at  $x = 0$  ?
- Find all values of the parameter 'a' for which the inequality  $4^x - a2^x - a + 3 \leq 0$  is satisfied by at least one real  $x$ .
- |   |                     |
|---|---------------------|
| <b>Column - I</b>   | <b>Column - II</b>  |
| (A) $\lim_{x \rightarrow \pi/2} [\sin^{-1} \sin x] =$                                 | (p) -2              |
| (B) $\lim_{x \rightarrow -\infty} [\tan^{-1} x] =$                                    | (q) 0               |
| (C) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x} =$ | (r) 1               |
| (D) $\lim_{x \rightarrow 0^+} \left[ \frac{\sin x }{x} \right] =$                     | (s) does not exist. |

( $\therefore$  where  $[ \cdot ]$  denotes greatest integer function)



# Answers Key

1. (C)    2. (A)    3. (A)    4. (A)

5. (i) 1    (ii)  $\frac{1}{2}$     6.  $a = 1, a = \frac{-1}{4}$

7.  $a \in [2, \infty)$     8.  $(A) \rightarrow r, (B) \rightarrow p, (C) \rightarrow s, (D) \rightarrow q$

